The Largest Scale We Can Detect in the Universe and the Inflation

Wen Zhao, Yang Zhang
Astrophysics Center
University of Science and Technology of China
Hefei, Anhui, China

Abstract

From the damping of the Cosmic Microwave Background Radiation (CMB) anisotropy power spectrum at large scale and the recent accelerating expansion of the Universe, we find that, there may be a largest scale which we can detect in the Universe. From this, we can get the inflation parameters as spectrum index n_s of the initial scalar spectrum, e-fold N, Hubble parameter H, the ratio of tensor and scalar r, the lasting time of reheating stage α for special inflation models. We do them in three inflation models, and find that all the results fit fairly well with the observations and the inflation theory.

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e-mail: wzhao7@mail.ustc.edu.cn

The release of the high resolution full sky Wilkinson Microwave Anisotropy Probe (WMAP) data shows that the data is consistent with the predictions of the standard inflation-ΛCDM cosmic model, expect for several puzzles[1][2][3][4][5][6][7]. One of those is that the power spectrum shows that there is a much damping at the large scale $l < 10$[2][6][7], which has been much deeply discussed[8]. The simplest explanation is that there is a cut at the very small wavenumber (IR cut-off) for the initial scalar perturbation. We show the power spectrum of CMB in figure (1), where the dots are the WMAP observation result, and the red line is the spectrum without IR cut-off, the green line with cut-off at 1/k=4000Mpc, the blue line with cut-off at 1/k=3000Mpc, the magenta line with cut-off at 1/k=2000Mpc. We have set the scale factor of now $a_0 = 1$, and chosen the cosmological parameters as $\Omega_m = 0.047$, $\Omega_\Lambda = 0.693$, $\Omega_{dm} = 0.29$, $h = 0.72$, $n_s = 0.99$[5], and without consider the reionization and the running of n_s. From this figure, we find that the cut of the initial perturbation is nearly at $1/k \simeq 2000 - 4000Mpc$, but why this cut-off exist?

We all know that there is an inflation stage at the very early Universe[9][10]in the inflation-ΛCDM cosmic model. The attraction of this paradigm is that it can set the initial conditions for the subsequent hot big bang, which otherwise have to be imposed by hand. One of these is that there be no unwanted relics (particles or topological defects which survive to the present and contradict observation). Another is that the initial density parameter should have the value $\Omega = 1$ to very accuracy, to ensure that its present value has at least roughly this value. There is also the requirement that the Universe be homogeneous and isotropic to high accuracy. But the most important is that the scale-invariant initial scalar perturbation power spectrum which predicted, has been detected from CMB and LSS, especially the recent WMAP[2][5][7] and SDSS[11] observation.

In this paradigm, the scale factor expanded much more rapidly in the initial inflation stage than the horizon[1]. From the sketch figure (2), we find that the scale of a/k_1 goes out the horizon at inflation, and reenteres the the horizon at radiation (dust)-dominating stage. Where k_0 is the smallest wavenumber, which had been in the horizon, when $k < k_0$, the wave had never been in the horizon, so the initial power spectrum should only at $k > k_0$. So for the scale with $l_2 = 2\pi a_i/k_2$ is larger that $l_H = 2\pi a_i/k_0$ (a_i is the scale factor at t_i, k_0 satisfies that $a_i/k_0 = 1/H$, where H is the Hubble parameter of inflation), the Universe will never be homogeneous and isotropic. if this cut of k_0 is just nearly $1/k_0 \sim 3000Mpc$, it will naturally answer the damping of the CMB power spectrum. The discussion in below will all base on this idea. If the Universe was dominated by dust, the scale factor $\propto t^{2/3}$, but the horizon is nearly $\propto ct$ (c is the speed of light), so there must be a time t_m, $2\pi a_m/k_0 \sim t_m$, after which, the Universe will not be homogeneous and isotropic. But that was not the true Universe.

A wide range of observational evidence indicates that our Universe may be accelerating expansion[12] [11][13][2][5][15][14]. If we assume that long-range gravity obeys Einstein's General Relativity, this suggests

[1]The horizon in this paper all denotes the particle horizon.

that most of our Universe is in some form of smooth dark energy with $\omega \simeq -1$, which can comprise $\sim 70\%$ of the critical energy. In this accelerating expansion Universe, there must be a time t_a, after which the scale factor would expand fast than the horizon. At this point, the wave which satisfied that $2\pi a_a/k_a = l_a$ (where a_a is the scale factor , and l_a is the horizon at t_a), exactly reenter the horizon, and immediately went out, so this is the largest scale which can reenter the horizon. It is also the largest scale which we can detect in the Universe.

For the flat Robertson-Walker metric

$$ds^2 = -c^2dt^2 - a^2(dx^2 + dy^2 + dz^2), \tag{1}$$

the time t_a is nearly the time when the Universe began to accelerating expansion, so $d^2a/dt^2 = 0$ at time $t = t_a$, which means that $\omega = \Sigma p_i/\Sigma\rho_i = -1/3$. For now the Universe is made of $\Omega_m = 0.047$, $\Omega_{dm} = 0.29$, $\Omega_{de} = 0.693$, and $\omega_{de} = -1$, one can get $z_1 = 0.653$. The particle horizon at time t is defined as:

$$l(t) = a(t)\int_0^t \frac{dt'}{a(t')}, \tag{2}$$

where the scale factor a can be get from the Friedmann equations. From which one can get the horizon at this time is $l_a = 6173Mpc$, where we have used the the Hubble constant $h = 0.72$. So $1/k_a = 1625Mpc$. We are surprised to find that k_a is nearly equate to k_0, which we have get from before. We know $1/k_0$ is the largest scale which had been in the horizon at inflation stage, so only if when the scale is smaller than which, the Universe can keep homogeneous and isotropic. But $1/k_1$ is the largest scale which can reenter the horizon in the accelerating Universe, so if the Universe is always homogeneous and isotropic, which requires that $k_0 \leq k_a$ (from before we know that it is satisfied), so we know that this accelerating expansion Universe will always be homogeneous and isotropic even if the inflation only expanded a period of time. This is one of the most important difference between the accelerating Universe and the decelerating Universe. If we can accept the below Assumption: assume the exact state that $k_0 \equiv k_a = k_0$ (from before, we find k_0 is a litter smaller than k_a, which is for we considering the inflation and dark energy in a crud way with H=constant, $\Omega_{de} = -1$, which all can affect k_0 and k_a to form this difference between them), which will mean that $1/k_a$ is always the largest scale we can detect in the Universe. The evolution of scale factor and horizon with time are shown in the sketch figure 2, where the black (solid) line is the particle horizon, the red (dash) line denotes the evolution of a/k_1, the blue (dot) line denotes a/k_0, and the magenta (dash dot) line denotes a/k_3, where $k_1 > k_0 > k_2$. We will discuss if this Assumption is rational in below.

With this Assumption, we can get the inflation parameters: the index of initial spectrum n_s, e-fold N, Hubble parameter of inflation H, the ratio of tensor and scalar perturbation r, and the expansion times of the scale factor in the reheating stage α for the special inflation models.

For

$$r = \frac{\Delta_h^2}{\Delta_R^2}, \tag{3}$$

where the scalar perturbation power spectrum is[7][5]

$$\Delta_R^2 = 2.95 \times 10^{-9} A, \tag{4}$$

The tensor power spectrum[16]

$$\Delta_h^2 = C^2(\mu)\frac{4}{\pi}\frac{H^2}{m_p^2}, \tag{5}$$

one can get the first relation

$$\frac{H}{m_p} = (\frac{2.95 \times 10^{-9} A r}{4C^2/\pi})^{1/2}. \tag{6}$$

where A is the scalar power spectrum amplitude, which is nearly 0.9 from WMAP result, and the constant number $C(\mu) \simeq 1$, $m_p = 2.4 \times 10^{18} GeV$ is the Planck energy scale.

If the inflation ended at time t_e. The expansion times of the scale factor between t_i and t_e is $a_e/a_i = e^N$, where N is the e-fold of the inflation. We can easily get another formula $a_a/a_e = \alpha H/T_a$, where T_a is the CMB temperature at time t_a, α was defined as:

$$\alpha = a_r/a_{re}, \tag{7}$$

where a_r and a_{re} are the scale factor at the end of the reheating stage and the beginning of the reheating stage. Here we have used that $aT = constant$ in the Universe. From the Assumption as before, one can get the second relation:

$$e^N = \alpha H/T_1 = 2.2 \times 10^{12} \alpha (H/GeV). \tag{8}$$

From $a_i/k_i = 1/H$, $2\pi a_1/k_1 = l_1$, and $k_i = k_1$, one can get the third relation:

$$a_1/a_i = e^{2N} = l_1/l_H = 2.1 \times 10^{42} (H/GeV). \tag{9}$$

From the relation of (8) and (9), one can get a simple constraint on the Hubble parameter of the inflation:

$$H < 4.3 \times 10^{17} GeV \equiv H_m, \tag{10}$$

for $\alpha > 1$, which is only dependent on the Assumption as before but not on the inflation models. For the special inflation cosmic models, one can get another two relations about N & n_s and r & n_s, here we consider three inflation models:

(1) $\lambda\phi^4$ inflation model: In this model, we have two simple relations of

$$N = \frac{3}{1-n_s}, \qquad r = \frac{16}{3}(1-n_s), \tag{11}$$

combining which with the relations (6), (8), (9), one can get the inflation parameters as below:

$$n_s = 0.95, \quad N = 64.53, \quad r = 0.25, \quad \alpha = 88.42, \quad H = 5.46 \times 10^{13} GeV < H_m; \tag{12}$$

(2) $V(\phi) = \Lambda^4(\phi/\mu)^2$ inflation model: In this model, two simple relations are

$$N = \frac{2}{1-n_s}, \qquad r = 4(1-n_s), \tag{13}$$

so we can get the inflation parameters in the same way:

$$n_s = 0.97, \quad N = 64.35, \quad r = 0.12, \quad \alpha = 105.55, \quad H = 3.86 \times 10^{13} GeV < H_m; \tag{14}$$

(3) $V(\phi) = \Lambda^4(\phi/\mu)^4$ inflation model: In this model, the two simple relations are

$$N = \frac{3}{1-n_s}, \qquad r = \frac{16}{3}(1-n_s), \tag{15}$$

so the inflation parameters are:

$$n_s = 0.95, \quad N = 64.53, \quad r = 0.25, \quad \alpha = 88.42, \quad H = 5.46 \times 10^{13} GeV < H_m; \tag{16}$$

From the observation, one gets the index of the initial spectrum of $n_s = 0.99 \pm 0.04$ from WMAP only, and $n_s = 0.96 \pm 0.02$ from WMAPext+2dFGRS+Lyman α[5]. The index from these three models are all fit fairly well with the observation. The constraint on r form WMAP and SDSS is $r < 0.36$[7][17], which is also consistent with the calculation results. Naturally, we accept that $N \geq 60$, $H \sim 10^{13} GeV$[7], which are all consistent with the models results. We also get the α is nearly 100 (independent on the reheating models), which tell us that the reheating process is a very quick process, at this stage, the scale factor only expanded nearly 100 times. From the before calculation, we find that, there inflation parameters are only dependent on the two relations of N & n_s, and r & n_s. The models of (1) and (3) have the same relations, so they give the same inflation parameters.

In summary, from the damping of the CMB anisotropy power spectrum at large scale, we think it is for that the initial power spectrum has a cut-off at $k < k_0$, which generated for the inflation must began at some time. One find that this cut-off is nearly equal to the largest scale which can reenter the horizon in the accelerating Universe. From this we elicit a simple Assumption: $k_0 \equiv k_a$ so the largest scale, which can be in the particle horizon in the inflation-accelerating expansion Universe is $\sim 1/k_a$. This Assumption will keep that the Universe will always be homogeneous and isotropic. To check the rationality of this Assumption, we calculated the inflation parameters od n_s, N, H,α in three inflation models from this Assumption, and find that they are all consistent with the observation and the inflation theory.

The accelerating expansion of the recent Universe is a very puzzle for cosmologist. Here we consider this question in another point: for the early inflation stage must have a beginning at some time, which make that there exist a largest scale $1/k_0$, when the scale is larger than it, the Universe will not be homogeneous and isotropic. The Universe began to decelerating expansion after inflation, if this deed was kept for all time, which will make the Universe not be homogeneous and isotropic at large scale at some time, for the expansion of the horizon is much quick that the scale factor. But to our surprise is that the Universe began to accelerating expansion at redshift $z \simeq 0.653$, which exactly elegantly make the Universe always be homogeneous and isotropic. Usually, we always ask the question: why the accelerating expansion exist? but here, we ask another question: why it is needed? Our answer is that: to keep the cosmological principle being right for all time. About why the Universe is this? which however, need to research. But from the discussion above, we think the below viewpoint is rational: the recent accelerating

must have some relation with the early inflation, these two accelerating expansion stages make Universe always be homogeneous and isotropic.

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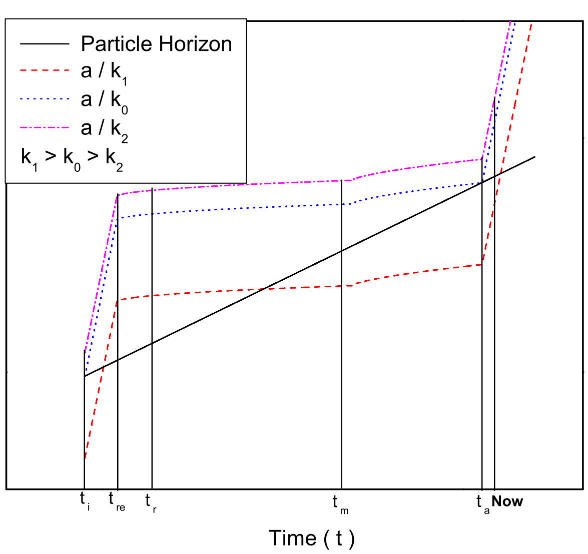

Particle Horizon
a / k1
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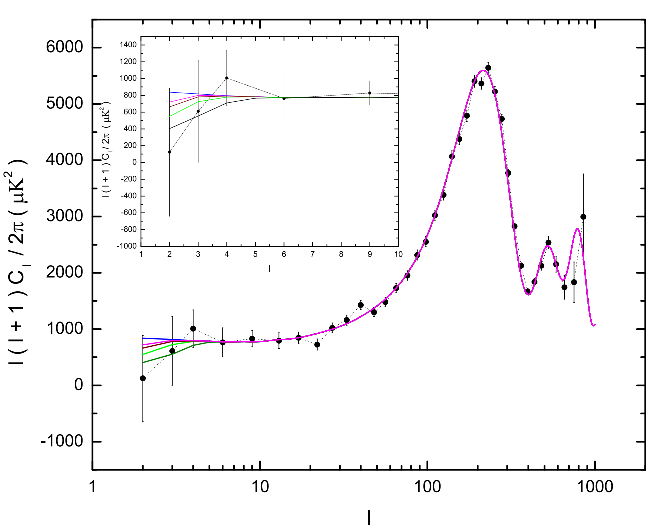

l (l + 1) C$_l$ / 2π (μK^2)
l